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LETTER TO THE EDITOR

Phonon emission anisotropy in a two-dimensional electron gas in crossed electric and magnetic fields

Keith A Benedict
Department of Physics, The University of Nottingham, Nottingham NG7 2RD, UK

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Abstract. Previous work on the angular distribution of phonons emitted by a heated two-dimensional electron gas is extended to include the effects of an in-plane (Hall) electric field such as would be present in a device in which the heating is due to the passage of an electric current. The anisotropy is seen to be such that there is an enhancement of the phonon emission in the direction of current flow.

Much recent work (see Challis *et al* 1991) has been carried out using the bulk phonons in a semiconductor device to probe the properties of a two-dimensional electron gas (2DEG) formed at an inversion layer, heterojunction or quantum well within the device. One method has been to heat the 2DEG by passing a current through it: the electrons then lose heat to the lattice by emitting acoustic phonons which travel ballistically to the surface of the crystal; hence, by measuring the local temperature on the surface of the device, the distribution in reciprocal space of emitted phonons (modulo focussing effects due to crystalline anisotropy) can be obtained (Kent *et al* 1990). The deviation of this from the isotropic distribution which would be produced by a simple black body contains information on the electronic structure of the 2DEG. A calculation by Toombs *et al* (1987) of the angular distribution of emitted phonons from a heated 2DEG in a strong perpendicular magnetic field has been extended (Benedict 1991) to include the effects of disorder. The purpose of this letter is to extend the former calculation to include the effects of the weak (Hall) electric field which gives rise to the heating current and to predict the form of the in-plane anisotropy due to the presence of the field. We review the basic ideas and notation used and describe the calculation of the angular distribution function for the case of an ideal 2DEG in a strong perpendicular magnetic field with a weak in-plane electric field and we then give a brief summary.

The notation and method of calculating the angular distribution of emitted phonons follows that of Benedict (1991, hereafter referred to as I). The angular distribution function $W_s(\theta,\phi)$ gives the rate of energy emitted as phonons of polarization s (s = LA, TA_{slow}, TA_{fast}) per unit time into a unit solid angle oriented in the direction specified by the polar angles θ and ϕ, the relevant polar axis being the normal to the 2DEG. In section 3 of I it was shown that this function could be written in the form

$$W_s(\theta,\phi) = \frac{1}{4\pi^2}\int_0^\infty \mathrm{d}Q\ Q^2\omega_s(Q)\Lambda_s(Q,\theta)\Gamma(\omega_s(Q),q) \quad (1)$$

where Λ depends only on the properties of the lattice and Γ is a structure function for the 2DEG which can be written in the form

$$\Gamma(\omega, q) = \sum_{\alpha,\beta} |\langle\alpha|e^{iq\cdot r}|\beta\rangle|^2 f(\epsilon_\alpha)(1 - f(\epsilon_\beta))\delta(\epsilon_\alpha - \epsilon_\beta - \hbar\omega) \tag{2}$$

where α and β label the single-particle eigenstates of the 2DEG and f is the usual Fermi distribution.

The Hamiltonian for a 2DEG in a strong perpendicular magnetic field with an in-plane electric field has the form

$$H = (1/2m)(p - eA(r))^2 - eE \cdot r \tag{3}$$

choosing the x axis to be in the direction of the electric field and the Landau gauge $A = B(0, x)$ for the magnetic vector potential yields the well known simultaneous eigenstates of H and p_y

$$\langle x, y|n, k\rangle = \frac{1}{\sqrt{Ll_c}} e^{iky} \chi_n((x - x_0(k))/l_c) \tag{4}$$

with eigenvalues

$$\epsilon_{n,k} = (n + 1/2)\hbar\omega_c - v_d\hbar k - mv_d^2/2 = \epsilon_n^0 - v_d\hbar k \tag{5}$$

and $k = 2\pi m/L$ where $l_c = \sqrt{\hbar/eB}$ is the cyclotron length, $\omega_c = eB/m$ is the cyclotron frequency, $v_d = E/B$ is the classical drift velocity for electrons in crossed electric and magnetic fields, L is the length of the system, χ_n is the nth-harmonic oscillator wavefunction and $x_0(k) = l_c^2 k - v_d/\omega_c$ is the 'guiding centre' coordinate, restricted to lie in the range $-L/2 < x_0 < L/2$. Using these states in the above expressions gives the following form for the structure factor (henceforward magnetic units in which $\hbar = l_c = \omega_c = 1$ will be used):

$$\Gamma(\omega, q) = \frac{1}{2\pi} \sum_{n,m} U_{n,m}(q^2/2)\gamma_n(\omega)\delta(m - v_d q_y - \omega) \tag{6}$$

where $U_{n,m}$ is the matrix element used in I and $\gamma_n(\omega)$ is given by

$$\gamma_n(\omega) = \sum_k f(\epsilon_{n,k} + \omega)(1 - f(\epsilon_{n,k})) = \frac{e\Delta V}{2\pi v_d^2} \int_{\epsilon_n^0 - e\Delta V}^{\epsilon_n^0} d\xi\, f(\xi + \omega)(1 - f(\xi)) \tag{7}$$

where ΔV is the total voltage drop across the sample.

The most important term in the expression for the structure function, from the point of view of the in-plane anisotropy (i.e. ϕ dependence) is the Dirac delta function; the origin of this constraint can be understood by reference to figure 1 which shows the energy levels of the 2DEG. The diagram shows a transition from an initial state X in a Landau level above the chemical potential to a final state W below the chemical potential resulting in the emission of a phonon (the edge states, which are localized to a region of size $\sqrt{n}l_c$ of the sample boundary where n is the Landau level

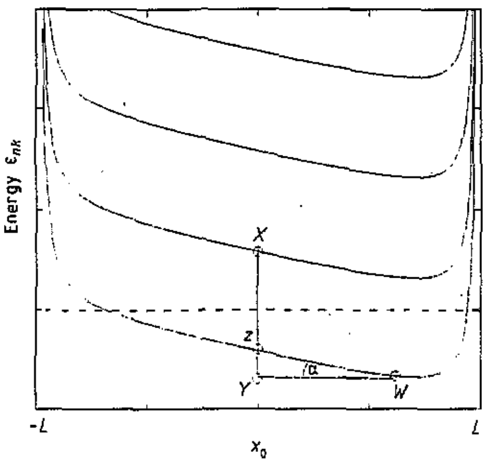

Figure 1. The energy level structure of an ideal 2DEG in a strong perpendicular magnetic field with an in-plane electric field; the horizontal axis is the guiding centre coordinate x_0, and the transition marked illustrates the origin of the constraint represented by the delta function in equation (6).

index, are neglected in this treatment). The distance XY is $\omega = c_s Q$, the distance XZ is 1 ($= \hbar\omega_c$ in conventional units), the distance WY is $q_y = Q \sin\theta \sin\phi$ and the slope of the dispersion curve is $\tan\alpha = v_d$ the drift velocity; hence, by elementary trigonometry, $v_d Q \sin\theta \sin\phi = 1 - c_s Q$.

Substituting this form of Γ (as well as the form of Λ appropriate for the deformation potential interaction and assuming a perfectly two-dimensional electron gas) into the expression for the angular distribution function gives

$$W_s(\theta, \phi) \sim \sum_{n,m} Q_{m,s}(\theta, \phi)^5 U_{n,m}(Q_{m,s}^2 \sin^2\theta/2)\gamma_n(c_s Q_{m,s}) \qquad (8)$$

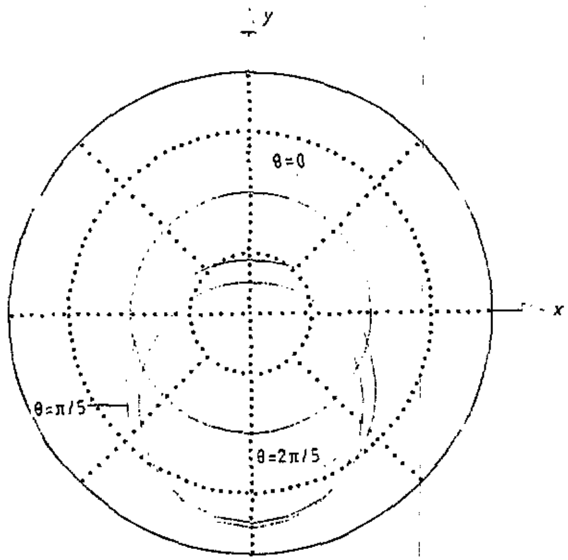

Figure 2. A polar plot of $Q_1^7(\theta, \phi)e^{-Q_1^2 \sin^2\theta/2}$ against ϕ for values of $\theta = 0, \pi/5, 2\pi/5$; as the eccentricity of the curve and the distance of its centroid from the origin increase with increasing θ, the value of $v_d/c_s = 0.25$ was used.

where

$$Q_{m,s}(\theta,\phi) = \frac{m}{c_s}\frac{1}{1+(v_d/c_s)\sin\theta\sin\phi}. \qquad (9)$$

From this it is clear that for weak electric fields the relevant small parameter controlling the in-plane anisotropy is the ratio of the drift velocity v_d to the speed of sound c_s. In order to illuminate this, the critical electric field at which $v_d = c_s$ for a silicon [100] 2DEG in a field of 5 T for the LA mode is roughly 4.5×10^4 V m^{-1}. The dominant ϕ dependence is then of the form $W(\phi) \sim Q_1^{-\nu} e^{-Q_1^2 \sin^2\theta/2}$ where ν is an m-dependent power ($\nu = 7$ for the most important case $m = 1$). This is shown on a polar plot in figure 2 for a range of values of θ.

From the calculation described above it can be seen that the in-plane anisotropy caused by the electric field is such that the phonon emission is enhanced in the direction of current flow, i.e. in the negative y direction and reduced in the opposite direction. The polar plot of figure 2 is approximately an ellipse with eccentricity $\nu v_d/c_s$, so the degree of anisotropy is directly related to the strength of the electric field. It is hoped that, given the detailed knowledge that now exists about the phonon focussing effect, the anisotropy could be experimentally measured in silicon devices using the imaging techniques described in Kent *et al.*

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References

Benedict K A 1991 *J. Phys.: Condens. Matter* **3** 1279

Challis L J, Kent A J and Rampton V W 1991 *Semicond. Sci. Technol.* **5** 1179

Kent A J, Hardy G A, Hawker P and Hurley D C 1990 *Phonons 89: Proc. 3rd Int. Conf. on Phonon Physics (Heidelberg, 1990)* vol 2, ed S Hunklinger, W Ludwig and G Weiss (Singapore: World Scientific) p 1010

Toombs G A, Sheard F W, Nielson D and Challis L J 1987 *Solid State Commun* **64** 577